Investigation • Bugs, Bugs, Everywhere Bugs

Name ______________________________ **Period** __________ **Date** ____________

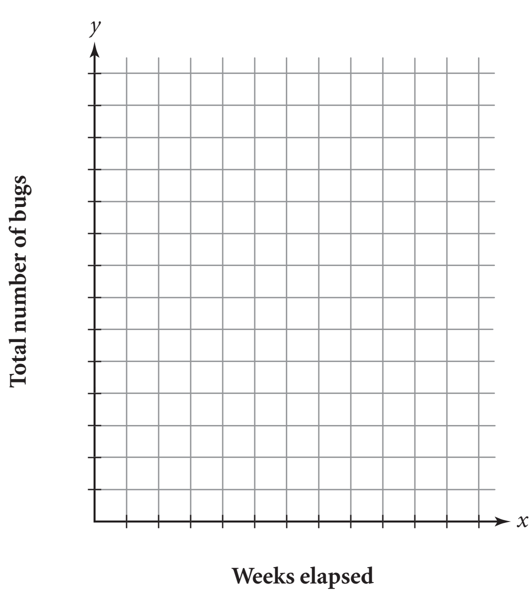
Imagine that a bug population has invaded your classroom. One day you notice 16 bugs. Every day new bugs hatch, increasing the population by 50% each week. So, in the first week the population increases by 8 bugs.

Step 1 In the table record the total number of bugs at the end of each week for 4 weeks.

Weeks elapsed	Total number of bugs	Increase in number of bugs (rate of change per week)	Ratio of this week's total to last week's total
Start (0)	16		
1		8	
2			
3			
4			

Step 2 The increase in the number of bugs each week is the population's rate of change per week. Calculate each rate of change and record it in your table. Does the rate of change show a linear pattern? Why or why not?

Step 3 Let x represent the number of weeks elapsed, and let y represent the total number of bugs. Graph the data using (0, 16) for the first point. Connect the points with line segments and describe how the slope changes from point to point.



Step 4 Calculate the ratio of the number of bugs each week to the number of bugs the previous week, and record it in the table. For example, divide the population after 1 week has elapsed by the population when 0 weeks have elapsed. Repeat this process to complete your table. How do these ratios compare? Explain what the ratios tell you about the bug population growth.

Step 5 What is the **constant multiplier** for the bug population? How can you use this number to calculate the population when 5 weeks have elapsed?

Step 6 Model the population growth by writing a recursive routine. [▶ See **Calculator Note 3A: Recursion on a List** to review recursive routines. ◀] Describe what each part of this calculator command does.

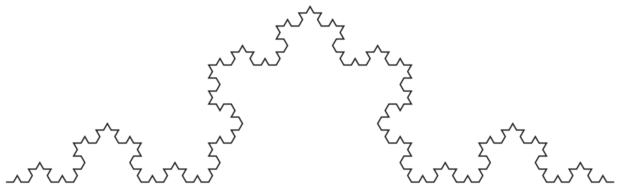
Step 7 Press ENTER a few times to check that your recursive routine gives the sequence of values in the column "Total number of bugs" in your table. Use the routine to find the bug population at the end of weeks 5 to 8.

Step 8 What is the bug population after 20 weeks have elapsed? After 30 weeks have elapsed? What happens in the long run?

Investigation • Growth of the Koch Curve

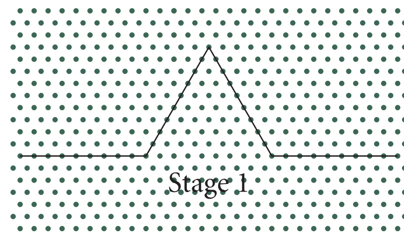
Name ______________________________ **Period** __________ **Date** ______________

You will need: the worksheet Growth of the Koch Curve



In this investigation you will look for patterns in the growth of a fractal called the *Koch curve*. Here you will think about the relationship between the length of the Koch curve and repeated multiplication. Stage 0 of the Koch curve is already drawn on the worksheet. It is a segment 27 units long.

Step 1 Draw the Stage 1 figure below the Stage 0 figure. The first segment is drawn for you on the worksheet. As shown here, the Stage 1 figure has four segments, each $\frac{1}{3}$ the length of the Stage 0 segment.



Step 2 Determine the total length at Stage 1 and record it in this table.

Stage number	Total length (units)	Ratio of this stage's length to previous stage's length
0	27	
1		
2		
3		

Step 3 Draw the Stage 2 and 3 figures. The first segment for each stage is drawn for you. Record the total length at each stage.

Step 4 Find the ratio of the total length at any stage to the total length at the previous stage. What is the constant multiplier? Record your ratios in the table.

Step 5 Use your constant multiplier from Step 4 to predict the total lengths of this fractal at Stages 4 and 5.

Step 6 How many times do you multiply the original length at Stage 0 by the constant multiplier to get the length at Stage 2? Write an expression that calculates the length at Stage 2.

Step 7 How many times do you multiply the length at Stage 0 by the constant multiplier to get the length at Stage 3? Write an expression that calculates the length at Stage 3.

Step 8 If your expressions in Steps 6 and 7 do not use exponents, rewrite them so that they do.

Step 9 Use an exponent to write an expression that predicts the total length of the Stage 5 figure. Evaluate this expression using your calculator. Is the result the same as you predicted in Step 5?

Step 10 Let x represent the stage number, and let y represent the total length. Write an equation to model the total length of this fractal at any stage. Graph your equation on your calculator and check that the calculator table contains the same values as your table.

Step 11 What does the graph tell you about the growth of the Koch curve?

Investigation • Moving Ahead

Name ______________________________ **Period** __________ **Date** ____________

Step 1 Rewrite each product below in expanded form, and then rewrite it in exponential form with a single base. Use your calculator to check your answers.

a. $3^4 \cdot 3^2$

b. $x^3 \cdot x^5$

c. $(1 + 0.05)^2 \cdot (1 + 0.05)^4$

d. $10^3 \cdot 10^6$

Step 2 Compare the exponents in each final expression you got in Step 1 to the exponents in the original product. Describe a way to find the exponents in the final expression without using expanded form.

Step 3 Generalize your observations in Step 2 by filling in the blank.

$b^m \cdot b^n = b^{\square}$

Step 4 Apply what you have discovered about multiplying expressions with exponents.

a. The number of ants in a colony after 5 weeks is $16(1 + 0.5)^5$. What does the expression $16(1 + 0.5)^5 \cdot (1 + 0.5)^3$ mean in this situation? Rewrite the expression with a single exponent.

b. The depreciating value of a truck after 7 years is $11{,}500(1 - 0.2)^7$. What does the expression $11{,}500(1 - 0.2)^7 \cdot (1 - 0.2)^2$ mean in this situation? Rewrite the expression with a single exponent.

c. The expression $A(1 + r)^n$ can model n time periods of exponential growth. What does the expression $A(1 + r)^{n+m}$ model?

Step 5 How does looking ahead in time with an exponential model relate to multiplying expressions with exponents?

Investigation • A Scientific Quandary

Name ______________________________ **Period** __________ **Date** ______________

Consider these two lists of numbers:

In scientific notation	**Not in scientific notation**
3.4×10^5	27×10^4
7.04×10^3	120,000,000
6.023×10^{17}	42.682×10^{29}
8×10^1	4.2×12^6
1.6×10^2	$4^2 \times 10^2$

Step 1 Classify each of these numbers as in scientific notation or not. If a number is not in scientific notation, tell why not.

a. 4.7×10^3 **b.** 32×10^5 **c.** $2^4 \times 10^6$

d. 1.107×10^{13} **e.** 0.28×10^{11}

Step 2 Define what it means for a number to be in scientific notation.

Use your calculator's scientific notation mode to help you figure out how to convert standard notation to scientific notation and vice versa.

Step 3 Set your calculator to scientific notation mode.
[▶ See **Calculator Note 6C: Scientific Notation.** ◀]

Step 4 Enter the number 5000 and press ENTER. Your calculator will display its version, 5×10^3. Use the table to record the standard notation for this number, 5000, and the equivalent scientific notation.

Standard notation	Scientific notation
5,000	
250	
−5530	
14,000	
7,000,000	
18	
−470,000	

Step 5 Using the table, repeat Step 4 for the additional numbers in standard notation. Record their scientific notations in the table.

Step 6 In scientific notation, how is the exponent on the 10 related to the number in standard notation? How are the digits before the 10 related to the number in standard notation? If the number in standard notation is negative, how does that show up in scientific notation?

Step 7 Write a set of instructions for converting 415,000,000 from standard notation to scientific notation.

Investigation • A Scientific Quandary (continued)

Step 8 Write a set of instructions for converting 6.4×10^5 from scientific notation to standard notation.

Investigation • The Division Property of Exponents

Name ____________________ **Period** __________ **Date** ______________

Step 1 Write the numerator and the denominator of each quotient in expanded form. Then reduce to eliminate common factors. Rewrite the factors that remain with exponents. Use your calculator to check your answers.

a. $\frac{5^9}{5^6}$ **b.** $\frac{3^3 \cdot 5^3}{3 \cdot 5^2}$ **c.** $\frac{4^4 x^6}{4^2 x^3}$

Step 2 Compare the exponents in each final expression you got in Step 1 to the exponents in the original quotient. Describe a way to find the exponents in the final expression without using expanded form.

Step 3 Use your method from Step 2 to rewrite this expression so that it is not a fraction. You can leave $\frac{0.08}{12}$ as a fraction.

$$\frac{5^{15}\left(1 + \frac{0.08}{12}\right)^{24}}{5^{11}\left(1 + \frac{0.08}{12}\right)^{18}}$$

Recall that exponential growth is related to repeated multiplication. When you look ahead in time you multiply by repeated constant multipliers, or increase the exponent. To look back in time you will need to undo some of the constant multipliers, or divide.

Step 4 Apply what you have discovered about dividing expressions with exponents.

a. After 7 years the balance in a savings account is $500(1 + 0.04)^7$. What does the expression $\frac{500(1 + 0.04)^7}{(1 + 0.04)^3}$ mean in this situation? Rewrite this expression with a single exponent.

b. After 9 years of depreciation, the value of a car is $21{,}300(1 - 0.12)^9$. What does the expression $\frac{21{,}300(1 - 0.12)^9}{(1 - 0.12)^5}$ mean in this situation? Rewrite this expression with a single exponent.

c. After 5 weeks the population of a bug colony is $32(1 + 0.50)^5$. Write a division expression to show the population 2 weeks earlier. Rewrite your expression with a single exponent.

d. The expression $A(1 + r)^n$ can model n time periods of exponential growth. What expression models the growth m time periods earlier?

Step 5 How does looking back in time with an exponential model relate to dividing expressions with exponents?

Investigation • More Exponents

Name ______________________ **Period** __________ **Date** ____________

Step 1 Use the division property of exponents to rewrite each of these expressions with a single exponent. Use your calculator to check your answers.

a. $\frac{y^7}{y^2}$ **b.** $\frac{3^2}{3^4}$ **c.** $\frac{7^4}{7^4}$ **d.** $\frac{2}{2^5}$ **e.** $\frac{x^3}{x^6}$

f. $\frac{z^8}{z}$ **g.** $\frac{2^3}{2^3}$ **h.** $\frac{x^5}{x^5}$ **i.** $\frac{m^6}{m^3}$ **j.** $\frac{5^3}{5^5}$

Some of your answers in Step 1 should have positive exponents, some should have negative exponents, and some should have a zero exponent.

Step 2 How can you tell what type of exponent will result simply by looking at the original expression?

Step 3 Go back to the expressions in Step 1 that resulted in a negative exponent. Write each in expanded form. Then reduce them.

Step 4 Compare your answers from Step 3 and Step 1. Tell what a base raised to a negative exponent means.

Step 5 Go back to the expressions in Step 1 that resulted in an exponent of zero. Write each in expanded form. Then reduce them.

Step 6 Compare your answers from Step 5 and Step 1. Tell what a base raised to an exponent of zero means.

Step 7 Use what you have learned about negative exponents to rewrite each of these expressions with positive exponents and only one fraction bar.

a. $\frac{5^{-2}}{1}$ **b.** $\frac{1}{3^{-8}}$ **c.** $\frac{4x^{-2}}{z^2y^{-5}}$

Step 8 In one or two sentences, explain how to rewrite a fraction with a negative exponent in the numerator or denominator as a fraction with positive exponents.

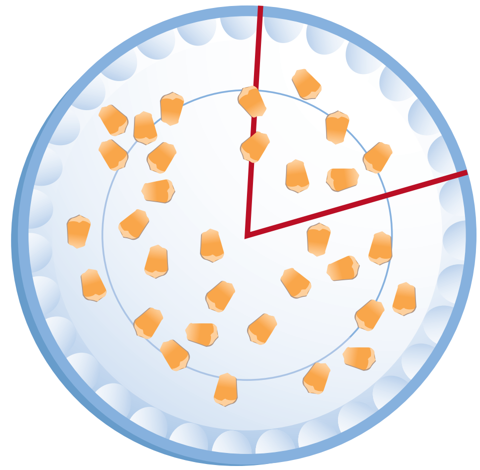
Investigation • Radioactive Decay

Name ______________________________ **Period** __________ **Date** ______________

You will need: a paper plate, a protractor, a supply of small counters

The particles that make up an atom of some elements, like uranium, are unstable. Over a period of time specific to the element, the particles will change so that the atom eventually will become a different element. This process is called **radioactive decay.**

In this investigation your counters represent atoms of a radioactive substance. Draw an angle from the center of your plate, as illustrated. Counters that fall inside the angle represent atoms that have decayed.



Step 1 Count the number of counters. Record this in the table as the number of "atoms" after 0 years of decay. Pick up all of the counters.

"Years" elapsed	"Atoms" remaining	Successive ratios

Procedure Note

Create a procedure for dropping counters randomly on the plate. Be sure that your method results in an approximately even distribution. Make a plan for handling counters that fall on the lines of your angle and those that miss the plate—they need to be accounted for too.

Step 2 Drop the counters on the plate. Count and remove the counters that fall inside the angle—these atoms have decayed. Subtract from the previous value and record the number remaining after 1 year of decay. Pick up the remaining counters.

Step 3 Repeat Step 2 until you have fewer than ten atoms that have not decayed. Each drop will represent another year of decay. Record the number of atoms remaining each time.

Step 4 Let x represent elapsed time in years, and let y represent the number of atoms remaining. Make a scatter plot of the table data on your calculator. What do you notice about the graph?

Step 5 Calculate the ratios of atoms remaining between successive years. That is, divide the number of atoms after 1 year by the number of atoms after 0 years; then divide the number of atoms after 2 years by the number of atoms after 1 year; and so on. Record the ratios in the table. How do the ratios compare?

Step 6 Choose one representative ratio. Explain how and why you made your choice.

Step 7 At what rate did your atoms decay?

Step 8 Write an exponential equation that models the relationship between time elapsed and the number of atoms remaining.

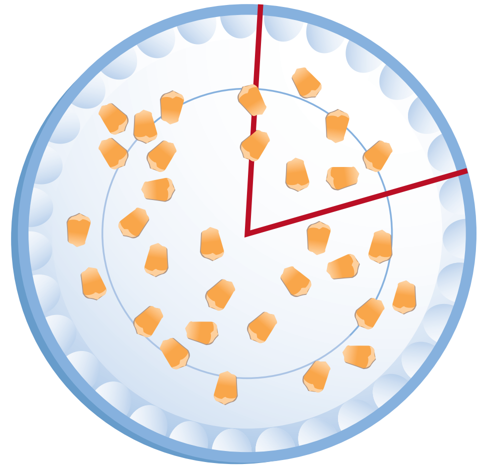
Step 9 Graph the equation with the scatter plot on your calculator. How well does it fit the data?

Step 10 If the equation does not fit well, which values could you try to adjust to give a better fit? Record your final equation when you are satisfied.

Step 11 Measure the angle on your plate. Describe a connection between your angle and the numbers in your equation.

Step 12 Based on what you've learned and the procedures outlined in this investigation, write an equation that would model the decay of 400 counters, using a central angle of 60°. What are some of the factors that might cause differences between actual data and values predicted by your equation?

Name ________________________ **Period** __________ **Date** ______________



The particles that make up an atom of some elements, like uranium, are unstable. Over a period of time specific to the element, the particles will change so that the atom eventually will become a different element. This process is called **radioactive decay.**

In this investigation counters represent atoms of a radioactive substance. An angle was drawn from the center of the plate, as illustrated. Counters that fall inside the angle represent atoms that have decayed.

Step 1 Objects were used as counters, and they were counted. This total was recorded in the table as the number of "atoms" after 0 years of decay.

"Years" elapsed	"Atoms" remaining	Successive ratios
0	201	
1	147	
2	120	
3	94	
4	71	
5	52	
6	42	
7	32	
8	28	
9	22	
10	18	
11	15	
12	12	
13	10	
14	9	

Step 2 The counters were dropped on the paper plate. Those that fell inside the angle were counted and removed—these represented atoms that had decayed. This number was subtracted from the previous value, and the result was recorded as the number remaining after 1 year of decay; in this case, the number remaining was 147. The remaining counters were picked up.

Step 3 Step 2 was repeated until fewer than ten atoms remained. Each drop represented another year of decay. All of these data were recorded in the table.

Step 4 Let x represent elapsed time in years, and let y represent the number of atoms remaining. Make a scatter plot of the table data on your calculator. What do you notice about the graph?

Step 5 Calculate the ratios of atoms remaining between successive years. That is, divide the number of atoms after 1 year by the number of atoms after 0 years; then divide the number of atoms after 2 years by the number of atoms after 1 year; and so on. Record the ratios in the table. How do the ratios compare?

Step 6 Choose one representative ratio. Explain how and why you made your choice.

Step 7 At what rate did the atoms decay?

Step 8 Write an exponential equation that models the relationship between time elapsed and the number of atoms remaining.

Step 9 Graph the equation with the scatter plot on your calculator. How well does it fit the data?

Step 10 If the equation does not fit well, which values could you try to adjust to give a better fit? Record your final equation when you are satisfied.

Step 11 The group that collected the data in the table used a plate with a 68° angle. Describe a connection between the angle and the numbers in your equation.

Step 12 Based on what you've learned and the procedures outlined in this investigation, write an equation that would model the decay of 400 counters, using a central angle of 60°. What are some of the factors that might cause differences between actual data and values predicted by your equation?